Experimental and Theoretical Studies on Loaded Circular Membranes

The Mrudanga produces near-harmonic overtones due to the Karane that modifies the series of inharmonic overtones of an unloaded membrane into a nearly harmonic series. This was first observed by Sir C.V. Raman who conducted a series of experiments in this regard. In practice however, and as several studies have shown, while the higher order overtones form a nearly harmonic series, the fundamental is actually out of tune and is typically about 1.07 times the true fundamental frequency and roughly corresponds to the note *Suddha Rishabha* (*Komal Rishab*). This ratio may also vary depending on the construction of the Karane and its material properties. The *Dheem* stroke corresponds to the mode (0, 1) where the whole membrane vibrates as a single unit and its frequency is higher than the required fundamental, as suggested by the harmonic overtones.

Several theoretical studies have been conducted by modeling the Mrudanga as a loaded circular membrane. In this context Prof. B.S. Ramakrishna's work assumes considerable significance. His theoretical model of the Mrudanga head as a "composite membrane" proves clearly the harmonicity of the overtones with the exception of the fundamental that is a bit high. There are several other studies as well, both theoretical and experimental, which clearly point to this non ideal behavior of the Mrudanga overtones. Rossing has presented several experimental studies on the Mrudanga and has demonstrated how the inharmonic overtones of the Mrudanga gradually move towards their harmonic slots as the Karane is built up layer by layer.

There are several other theoretical studies done in this regard. A further refinement of the two density model of Ramakrishna has been done by Hagues and Piette of Durham University, where the vibrating membrane is modeled as a 3 density system with the Karane having two densities instead of one. Interested readers who wish to learn more about the advanced modeling of the Mrudanga and Tabla may refer to some of the following materials:

1. C.V. Raman, *The Indian Musical Drums*, Proceedings of the Indian Academy of Sciences,1A, 179-188,1935.
2. B.S. Ramakrishna and M.M.Sondhi, *Vibrations of Indian Musical Drums Regarded as Composite Membranes*, Journal of the Acoustical Society of America, 26, 4, 523-529, 1954.
3. B.S. Ramakrishna, *Modes of Vibration of the Indian Drum Dagga or Left-hand Thabala*,
4. The Journal of Acoustical Society of America, Vol. 29, No. 2, 234-238, February, 1957.
5. N Fletcher and T. Rossing, *The Physics of Musical Instruments*, Springer-Verlag, New York, 1991.
6. T.D. Rossing, *Acoustics of Drums*, Physics Today, 40-47, March, 1992.
7. S.S. Malu and A. Siddharthan, *Acoustics of the Indian Drum*, arXiv:math-ph/0001030v1, February 6, 2008.
8. Sathej and R. Adhikari, *TheEigenspectra of Indian Musical Drums*, Journal of theAcoustical Society of America, 2, 831-838, 2009.